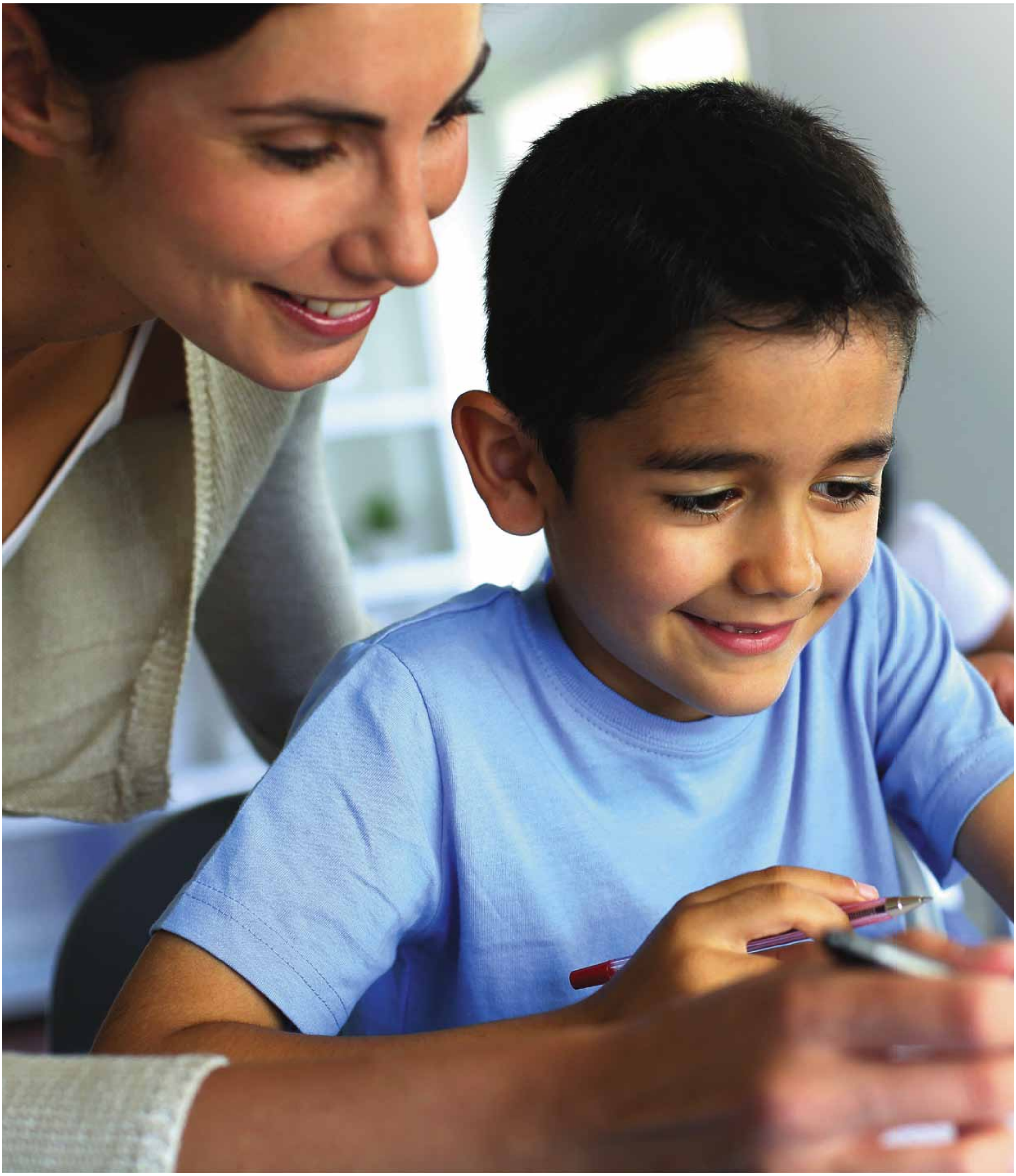

Assessing Basic Fact *Fluency*

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.

By Gina Kling and Jennifer M. Bay-Williams

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Think about how you assess reading fluency. Does your assessment plan involve listening and observing as children read as well as asking reading comprehension questions? Now imagine what you might learn about students' reading fluency if you used *only* timed quizzes. How would your confidence in your assessment change?

Formative assessments—including observations, interviews, performance tasks, and journaling—have become common practice in many classrooms, with a recognition that by using different ways to assess children, we gain a more comprehensive, accurate picture of what they know, what they do not know, and their misconceptions. These data are then used to design instruction accordingly (Wiliam 2011). Yet, in spite of this trend in other areas of education, timed, skill-based assessments continue to be the prevalent measure of basic mathematics facts achievement. As a result, many rich opportunities for assessing basic fact fluency are lost. In this article, we share a variety of ways to formatively assess basic fact fluency. We define fluency, raise some issues related to timed testing, and then share a collection of classroom-tested ideas for authentic fact fluency assessment.

Defining fluency

A variety of interpretations exist for what procedural fluency (in general) and basic fact fluency (specifically) mean. Fortunately, recent standards, research, and reports provide a unified vision of what these terms signify. The Common Core State Standards for Mathematics (CCSSM) document describes *procedural fluency* as "skill in carrying out procedures *flexibly*, *accurately*, *efficiently*, and *appropriately*" (CCSSI 2010, p. 6). Likewise, Baroody (2006) describes basic fact fluency as "the efficient, appropriate, and flexible application of single-digit calculation skills and . . . an essential aspect of mathematical proficiency" (p. 22). These definitions reflect what has been described for years in research and standards documents (e.g., NCTM 2000, 2006; NRC 2001) as well as CCSSM grade-level expectations related to basic facts (see **table 1**).

Notice that the CCSSM expectations use two key phrases; the first is to *fluently* add and subtract (or multiply and divide), and the second is to *know from memory* all sums (products) of two one-digit numbers. To assess basic fact *fluency*, all four tenets of fluency (flexibility, appropriate strategy use, efficiency, and accuracy) must be addressed. Additionally, assessments must provide data on which facts students *know from memory*. Timed tests are commonly used to supply such data—but how effective are they in doing so?

TABLE 1

Past mathematics documents—as well as current standards, studies, and reports—offer a unified vision of what procedural fluency means.

CCSSM standards that address fluency and mastery with basic facts (italics added)

Kindergarten K.OA.A.5	*Fluently* add and subtract within 5
Grade 1 1.OA.C.6	Add and subtract within 20, *demonstrating fluency* for addition and subtraction within 10. *Use strategies* such as counting on; making 10; decomposing a number leading to a 10; using the relationship between addition and subtraction; and creating equivalent but easier or known sums
Grade 2 2.OA.B.2	*Fluently* add and subtract within 20 using *mental strategies* (refer to 1.OA.6). By the end of grade 2, *know from memory* all sums of 2 one-digit numbers
Grade 3 3.OA.C.7	*Fluently* multiply and divide within 100, using *strategies* such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 / 5 = 8) or properties of operations. By the end of grade 3, *know from memory* all products of 2 one-digit numbers.

Limitations and risks of timed mathematics tests

Timed tests offer little insight about how flexible students are in their use of strategies or even which strategies a student selects. And evidence suggests that efficiency and accuracy may actually be negatively influenced by timed testing. A study of nearly 300 first graders found a negative correlation between timed testing and fact retrieval and number sense (Henry and Brown 2008). Children who were frequently exposed to timed testing demonstrated *lower* progress toward knowing facts from memory than their counterparts who had not experienced as many timed tests. In fact, growing evidence suggests that timed testing has a negative impact on students (Boaler 2012, Henry and Brown 2008, Ramirez et al. 2013). Surprisingly, the anxiety that many children experience over timed testing is unrelated to how well they do on the tests. Even high-achieving children share such concerns as, "I feel nervous. I know my facts, but this just scares me" (Boaler 2012). Math anxiety appears as early as first grade, and this anxiety does not correlate with reading achievement (Ramirez et al. 2013). In other words, children's anxiety is specific to mathematics, not general academic work. And the struggling learner is not the only one who experiences anxiety: Ramirez and his colleagues found that children demonstrating a higher use of "working memory" (i.e., those who tended to use mathematical strategies that were more sophisticated) experienced the most negative impact on achievement as a result of math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively influenced by timed testing.

Fortunately, children can learn facts effectively without the use of timed testing. In a longitudinal study of twenty second graders, Kling found that without any timed testing or other rote fact activities, by the end of the year, the children demonstrated automaticity with addition facts (solved within 3 seconds) 95 percent of the time. Interestingly, the children performed strategies (e.g., making ten or near doubles) so quickly that it was impossible to distinguish between highly efficient strategy application and “knowing from memory.” Since the beginning of first grade, fact practice for these children had involved (a) activities within textbook lessons (b) weekly fact games and (c) activities such as Quick Images with ten frames that were used to foster discussion around strategies (see Kling 2011, Bay-Williams and Kling in press). This research suggests that timed assessments and drill may not be necessary for children to achieve fact mastery.

If timed mathematics assessments have questionable value and potentially negative psychological, emotional, and educational impact, why are they still so frequently used? We commonly hear three reasons. First, fluency is interpreted as synonymous with speed. We have already addressed that fluency is more comprehensive than speed. Second, some feel that timed tests prepare children for high-stakes tests. The research shared here convincingly shows it may do the opposite. Third, timed tests are the only assessments widely available for assessing fluency of basic facts. As we seek to rectify this last concern, the remainder of this article shares methods for more comprehensively and appropriately assessing students’ basic fact fluency.

Using formative assessment strategies

With an eye on the aspects of fluency (accuracy, efficiency, flexibility, and appropriate strategy selection), we can use various assessment strategies to see what students know (and do not know) and determine what our next instructional steps might be. All are approaches we have used with children in grades 1–4, and when used in combination with one another, these methods provide a comprehensive picture of a student’s level of fact mastery.

1. Interviews

Interviews provide the extraordinary opportunity to hear children explain what they know about a topic in a discussion format, during which teachers can ask follow-up and clarifying questions (Hodges, Rose, and Hicks 2012; Van de Walle et al. 2014). The insights gained from listening to a child can be invaluable for planning individualized instruction or interventions. And insights from a selection of students can inform instruction for the whole class. Consider which aspects of fluency to address using the questions posed in each of the sample interview protocols (see **fig. 1**). We see the interview as a quick way to get at all four categories of fluency (as well as to see if a student just knows

FIGURE 1

Below are protocols for student interviews, which are a way to quickly assess all four categories of fluency and see if a student just knows a fact. Insights from a selection of interviews can inform instruction for the whole class.

Protocol A. Assess **fluency**

1. Write 4 × 5 on a card [*point at card*]. What does 4 × 5 mean?
2. What is the answer to 4 × 5?
3. How did you find the answer to 4 × 5? Could you find it another way?
4. If your friend was having trouble remembering this fact, what strategy might you suggest to him or her?

Protocol B. Assess **flexibility** and **strategy selection**

1. What is 8 + 5?
2. How can you use 8 + 2 to help you solve 8 + 5?

OR

1. How can you use 3 × 7 to solve 6 × 7?

Protocol C. Assess **use of appropriate strategy**
(adapted from Henry and Brown 2008)

Probes
What is 7 + 8?
How did you figure it out? [*Ask regardless of how quickly or accurately they solve the fact.*]

Codes

R	= Recall
A	= Automatic (within 3 seconds)
M10	= Making 10 Strategy
ND	= Near Doubles Strategy
D	= Some other derived fact strategy
CO	= Counting on
CA	= Counting all
MCA	= Modeling and counting all

the fact). *Accuracy* is assessed as soon as the student responds, and *efficiency* is observed on the basis of how long it takes a student to solve the fact. *Flexibility* and *appropriate strategy selection* are addressed by such follow-up prompts as, "How did you figure it out?" or "How could you use this strategy to solve this fact?" Codes, such as those suggested in Protocol C, can facilitate recording during an interview (see **fig. 1**).

Interviews need not be one-on-one, sit-down events. They can be quick exchanges in the midst of other activities. For example, as students are lining up, ask, "Aaron, what is six times seven? How did you figure it out?" Furthermore, interviews have an added benefit of allowing students the opportunity to self-correct. For example, during interview assessments with thirty-eight beginning first graders, Kling found that 54 percent of the time children self-corrected incorrect answers as they explained how they figured out the fact. Furthermore, the children's self-reported strategies (i.e., "I counted on my fingers" or "I just knew it") were consistent with what the interviewer was able to observe 97 percent of the time. This suggests the potential of interviews as highly reliable and informative assessment tools.

2. Observations

Observation is a natural part of teaching, and recognizing which strategies students know can supply valuable insights to help support students as they learn new strategies and tackle unknown facts. To create organized and accurate records of observations, a list of students and facts can be attached to a clipboard (see

TABLE 2

Codes can facilitate recording during an interview.

(a) Use an accuracy table to review students' progression with addition facts.

	Within 5	Foundational facts			Within 10	Within 20
Name/facts		0, 1, 2	Combinations that make 10	Doubles		
Nicholas						
Kayla						
Cynthia						
Robbie						
⋮						

(b) A table can show the frequency of addition fact strategy use at a glance.

Name/strategies	1 more/ 2 more	Combinations that make 10	Making 10	Doubles	Find 5s	Applies commutativity
Nicholas	+			+	+	+
Kayla	+			+	+	
Cynthia		+		+		+
Robbie	+	+	+			+
⋮						

MEAGHAN GORZENSKI

As students turn over cards, observe to see and hear how *efficient* each student is as well as whether he or she chose an *appropriate strategy* or just knew.

table 2a); or a list can be tracked on the basis of which strategies students use (see **table 2b**). Equipped with these charts, you can observe as students engage in facts games, such as mathematized versions of classic games of War, Go Fish, Concentration, Old Maid, and Memory (for other games to teach and review basic facts, see Forbringer and Fahsl 2010; Kamii and Anderson 2003; Van de Walle, Karp, and Bay-Williams 2013; Kling 2011; Bay-Williams and Kling, in press; Kling and Bay-Williams, in press).

A critical aspect of meaningful use of games is to ask students to tell their teammates both the answer *and* how they found it. As students turn over cards, observe to see and hear how *efficient* each student is as well as whether he or she chose an *appropriate strategy* or if they just knew. The teacher might observe, for example, that many students are more *efficient* at solving 5 + 3 than they are at 3 + 8. These students may "just know" facts within ten but may apply strategies for the facts that have sums over ten. Such insights gained through observation can help the teacher select appropriate activities for continued learning and practice.

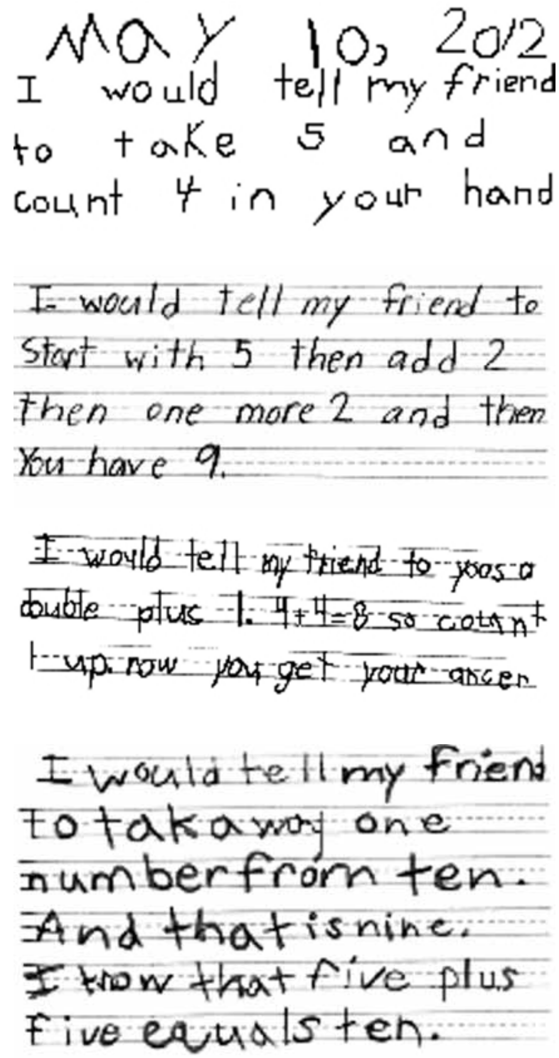
To enhance opportunities for assessing students during game play, consider having groups rotate through centers, stationing the teacher at one center and using probing questions, such as "How did you figure that out?" or "Are there any other ways you could figure it out?" One first-grade teacher had the following to say after using an observational checklist to formatively assess her students:

> This is an important tool that provides a more comprehensive check of which specific strategies a student has successfully mastered toward developing fluency with their basic facts. CCSSM provide specific strategies that students are expected to understand and use, and the chart provides me the opportunity to learn which strategies are being used effectively and where there are opportunities for further instruction and practice.

FIGURE 2

Various responses to a journal prompt illustrate the strategies that first graders used and reveal which children were able to appropriately select and explain an efficient strategy for the task.

If your friend did not know the answer to 4 + 5, how could he figure it out?



3. Journaling

Writing provides an excellent opportunity to assess flexibility and understanding of strategy selection and application. Children at any grade level can find ways to incorporate pictures,

> *Writing provides an excellent opportunity to assess flexibility and understanding of strategy selection and application. Children at any grade level can find ways to incorporate pictures, words, and numbers to communicate their strategies.*

words, and numbers to communicate their strategies. For example, **figure 2** shows a variety of first graders' responses to the journal prompt, "If your friend did not know the answer to 4 + 5, how could he figure it out?" Carefully review the responses, considering what they illustrate about the strategies used by the children. In contrast to what can be learned from a child's answer to 4 + 5 on a timed test, these samples offer rich opportunities to recognize which children can appropriately select and explain an efficient strategy for the task. This is important for deepening strategy understanding and also is reflected in the expectations of CCSSM and the related, forthcoming assessments. For example, Smarter Balanced Assessment Consortium (SBAC) lists the following as "evidence required" for grade 3. Note the application of strategies inherent in these expectations. The student—

- multiplies and divides facts *accurately*;
- multiplies and divides facts *using strategies*, such as the relationship between multiplication and division or properties of operations; and
- uses multiplication and division facts (SBAC 2012) (emphasis added).

TABLE 3

This collection of prompts addresses the four components of fluency with basic facts. Writing about their strategies on a weekly basis engages students in self-reflection and monitoring, as well as emphasizes the importance of strategies in practicing basic facts.

Writing prompts for developing fluency with the basic facts

Appropriate strategy selection	Flexibility
• Explain how to use the "count on" strategy for 3 + 9. • What strategy did you use to solve 6 + 8? • A friend is having trouble with some of his times 6 facts. What strategy might you teach him? • Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.	• How can you use 7 × 10 to find the answer to 7 × 9? • Solve 6 × 7 using one strategy. Now try solving it using a different strategy. • Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Does this strategy always work?
Efficiency	**Accuracy**
• What strategy did you use to solve 9 + 3? • How can you use 7 × 7 to solve 7 × 8? • Which facts do you "just know"? For which facts do you use a strategy?	• Crystal explains that 6 + 7 is 12. Is she correct? Explain how you know. • What is the answer to 7 × 8? How do you know it is correct (how might you check it)?

Creative writing ideas that address several components

- Develop a "Face the facts" or "Ask Cougar" column (like Dear Abby) for the class. (Pick a fun name for the column that makes sense for the class, such as the school mascot.) Students send a letter about a tough fact. Rotate different students into the role of responder. The responder writes letters back, suggesting a strategy for the tough fact.
- Create a strategy rhyme (e.g., If times four is giving me trouble, I'll remember to double and double).
- Make a facts survival guide. Children prepare pages illustrating with visuals (e.g., ten frames or arrays) of how find "tough" facts.
- Write a yearbook entry to some facts (e.g., Dear 8 × 7, I …)

(See McIntosh 1997 for many more ideas).

Meaningful writing tasks can be used across grade levels and operations. **Table 3** presents a collection of writing prompts that address the four components of fluency. Having an opportunity to write about strategies on a weekly basis engages students in self-reflection and self-monitoring as well as emphasizes the importance of strategies in practicing basic facts.

4. Quizzes

You may be surprised to see this section, given the major concerns raised earlier related to timed tests, but quizzes can be used effectively to assess efficiency as well as strategy use. Ensure that students "just know" their foundational facts before moving on to derived facts. Foundational facts are so named because they can be used to generate all the other facts using a strategy. Foundational facts in addition include one- and two-more-than, combinations that make ten, and doubles. For multiplication, they include × 1, × 2, × 5, and × 10. (See Kling and Bay-Williams, in press, for a discussion of foundational facts.) From these facts, we can derive all other facts. Quiz questions (see **fig. 3a**) can be used to see if students "just know" foundational facts.

Similarly, quizzes can be used to monitor facts that come more easily to students. For example, a quiz (see **fig. 3b**) assesses if students are recognizing the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies. The following adaptations can enhance the effectiveness of facts quizzes:

FIGURE 3

Quizzes that focus on fluency are alternatives to timed tests.

(a) Quiz questions can be used to see if students "just know" foundational facts.

Solve these problems and tell how you solved them.

4 × 5 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
10 × 6 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
6 × 2 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
5 × 3 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
2 × 9 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
3 × 10 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
5 × 7 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.
8 × 10 =	______	Check one: ______	I used this strategy: ______________
		______	I just knew.

(b) A quiz assesses if students recognize the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies.

On completion, say to class, "Circle the row that was easier for you to solve. If they were both the same, write 'same'."

Solve these addition problems.

ROW A:	9 + 1 =	8 + 1	5 + 1 =	3 + 1 =	6 + 1
ROW B:	1 + 8 =	1 + 7 =	1 + 4	1 + 2	1 + 9 =

GINA KLING

Adopt more flexibility and variety in how you assess students' basic fact fluency.

reflect and discuss

"Assessing Basic Fact *Fluency*"

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions, related to "Assessing Basic Fact *Fluency*" by Gina Kling and Jennifer M. Bay-Williams, are suggested prompts to aid you in reflecting on the article and on how the authors' ideas might benefit your own classroom practice. Consider the article independently and then discuss it with your colleagues.

Think of the basic facts assessments that you are currently using with respect to the following:

- Flexibility
- Efficiency
- Appropriate strategy use
- Accuracy

1. With your current assessments, what percentage of emphasis might you assign to each of the four categories above? Is this balance what you would like it to be? If not, how might you alter your assessments to equitably address the four areas of fluency?
2. As you reflect on your students' basic facts fluency, what would you like to know more about? Which of the assessment tools from the article might help you gain this knowledge? How might you use that assessment tool?
3. Discuss your reactions to the issue of timed tests. What might you do as a teacher or leader to avoid potential negative impacts of timed tests?
4. How might we help parents better understand fluency and help their children in the areas of flexibility and selecting appropriate strategies? How might you communicate the purpose of alternative assessment tools for basic facts with your students, parents, and school leadership?

Tell us how you used Reflect and Discuss as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to Teaching Children Mathematics *at* ***tcm@nctm.org****. Please include Readers Exchange in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit are subject to abridgment. Letters are also edited for style and content.*

- Choose one of the problems above and write about how you solved it.
- Tell which helper fact you used the most on this quiz.
- Circle facts you "just knew." Highlight those for which you used a strategy.
- Circle facts you are sure about. Draw a square around facts that took you longer to solve.

Meaningful fact assessment for teachers and students

We recognize that using timed tests is a deeply rooted practice for measuring basic fact mastery. We hope that we have effectively made a case for *why* this practice must change and *how* to make such a change. As the NCTM Assessment Principle states, "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (NCTM 2000, p. 11). Using the range of assessments described above accomplishes these goals, as they provide an opportunity for meaningful, targeted feedback to students that far exceeds the "right or wrong, fast or slow" feedback provided by timed testing. In fact, these assessments infuse a fifth and critical category of assessment: self-assessment. Interviews, journals, *and* quizzes on basic facts can and should encourage students to reflect on which facts and strategies they know well and which ones are tough for them. This self-assessment can be effectively followed up by having children identify and record strategies that could be used to efficiently determine the "tough" facts in the future. Over time, this self-assessment practice encourages children to instinctively apply effective strategies for challenging facts they encounter. As both teachers and students critique their growth with use of appropriate strategies, efficiency, flexibility, and accuracy, then *true* fluency with basic facts can become a reality for every child.

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Readers may be interested in "Research Suggests That Timed Tests Cause Math Anxiety" (p. 469), in which Jo Boaler makes a case for alternative assessment strategies.

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Download a free app for your smartphone. Then scan this code to access quizzes and observation checklists appended to this article at **www.nctm.org/tcm060**.