Applying UDL in Math

CAST has developed guidelines that are based in each of the three brain networks described in the Introduction to UDL. These guidelines have been made broad enough to apply to any subject. Here we highlight those that have a specific application to teaching mathematics.

Designing Instruction to Support Knowledge Networks: Multiple Means of Representation

Knowledge networks play a major role in the development of conceptual knowledge, which involves the linking of these representations in meaningful ways (Dehaene, 1997; Hasselbring & Moore, 1996; Hiebert & Lefevre, 1986; Smith & Katz, 1996; Thelen & Smith, 1994). Many components of mathematics rely on recognition networks. For example, students are expected to interpret and internalize models of numbers and operations. Recognizing spatial relations between objects is essential in many aspects of mathematics (van Garderen & Montague, 2003). Proficiency in algebra requires the ability to grasp the relationship between patterns in numbers and functions that represent them (NCTM, 1998; 2000). Additionally, conceptual understanding of mathematics is enhanced when learners have the opportunity to explore and manipulate representations (Eisenhart et al., 1993; Moreno & Mayer, 1999; Moyer, Niezgoda, & Stanley, 2005). Without a solid understanding of mathematical concepts, mathematics becomes a series of rote procedures (Ginsberg, 1997; Rittle-Johnson, Siegler, & Alibali, 2001; Wood & Sellers, 1997). Even the learning of basic facts is facilitated by a good understanding of number concepts (Crespo, Kyriakides, & McGee, 2005; Jordan, Hanich, & Kaplan, 2003)
The following guidelines describe ways to support knowledge networks when teaching mathematics.

- **Customize how information is presented**

Students may seem to have difficulty with math because of the way the content is presented. Math text pages often contain a myriad of text, illustrations, instructions, examples, and problems to solve that leave a student unsure of how to proceed. Simplifying the page layout and presenting less information on a page can help students to focus on the math. Other strategies that can help are using larger fonts, presenting fewer problems on a page, and providing lines or answer boxes for students to use. A special consideration is students with sensory deficits, such as blindness. Often the traditional formats are not appropriate for these students because they are inaccessible. In such cases, an alternative format is needed to represent concepts.

- **Define vocabulary and symbols; decode mathematical notation**

Math has specific language and symbols that students need to learn. Often words that are used commonly, such as difference, have specific meanings in math, and students need to know how to use them correctly. Other words, such as polynomial, are a part of a specialized math vocabulary. Additionally, math has its own set of symbols, and students need to understand their meanings. Students may need additional supports, such as math dictionaries and previews of new vocabulary, to help them understand the language of mathematics.

- **Illustrate key concepts with different representations**

Understanding mathematics requires recognizing and interpreting number concepts represented symbolically, linguistically, and in physical representations. For example, the concept that 5 is a larger number than 3 can be represented as

"five is greater than three" (linguistic)
5 > 3 (symbolic)
<u>1 2 3 4 5</u> (physical - number line)

Additionally, within each of these formats the concept can be represented in more than one way. The statement "three is less than five" is another representation, as is "3 < 5." Physically, we could represent the relationship between three and five using counting chips, Base 10 Blocks, Cuisenaire Rods, or a myriad of other materials, each of which has different properties. It is particularly important that students not only recognize mathematical concepts presented in each of these formats but that they recognize the relationship between them.

Although we often think that presenting a concept through a variety of media and formats is best, this is not always the case. As Howard Gardner points out, we should use a family of representations that are accurate and complementary. If the same concept is presented using different formats, it is important that the formats be chosen carefully to enhance that concept, so that the student will not view each as representing a different idea. When the formats are chosen carefully, using multiple media and formats can enhance learning.

When using a new or unfamiliar format, it is important to be sure that students grasp the underlying meaning and are able to connect this new representation to ones that they already know. What often seems transparent to us as adults is not at al clear to our students.

- **Provide or activate background knowledge; connect new knowledge to previously learned**

New knowledge is best learned by incorporating it into what has been previously learned. The NCTM Standards include one that explicitly states the importance of connecting new learning to what has been previously learned. Students need to understand applications of mathematical concepts to everyday life. They also need to understand the math they are learning within an overall framework of mathematical concepts. We want to create increasingly complex representations of mathematical content areas - a mathematical "web."

- **Highlight critical features, big ideas, and relationships**

Students learn structure and rules by recognizing critical elements and then generating and testing hypotheses about these elements. We need to select examples of the concepts we are teaching with care and make sure that students focus on the relevant features. Novice learners tend to focus on superficial elements, rather than the underlying structure, so, when introducing new information, we should explicitly highlight the significant structural features and, when we use a variety of formats, be sure that the students see the structural similarities. "Highlighting" also can be done through structured interaction with manipulative materials.

Examples should be carefully selected, based in students' current understanding of the concept, and used to extend it. If an example presents something from a new perspective, it should be clearly linked to one that the learner already understands.

– **Support transfer**

Often students learn math concepts in an isolated fashion; although they may be able to use them with familiar problems they do not see how to transfer their application to new situations. Students demonstrate a true understanding of math concepts only when they can apply them in a variety of situations. It is important to provide them with these novel situations and guide them to understand they relate to the more familiar ones.

Designing Instruction to Support Strategic Networks: Multiple Means of Action and Expression

Strategic networks include both networks that control routine actions or procedures and those we use to generate active strategies for solving problems. Strategic actions are heavily dependent on the goal and the context in which the actions will occur (Cooper, Shallice, & Faringdon, 1995; Dehaene, 1997; Jeannerod, 1997). In order to be an effective learner and problem-solver it is essential that basic skills and procedures become routine or automatic, allowing the learner to put more attention and effort into applying conceptual knowledge (Gersten & Chard, 1999; Hiebert & Lefevre, 1986; Jordan et al., 2003). However, the learner also must be able to select and order the appropriate procedures and to monitor their effectiveness in attaining a goal or solving a problem (Cary & Carlson, 1999; Hiebert & Lefevre, 1986; Lesh & Harel, 2003; Pressley, 1991) Students also need to know how to select appropriate strategies and to organize information effectively in order to solve complex problems (Jitendra, DiPipi, & Perron-Jones, 2002; Siegler, 2003).

Below are guidelines that illustrate ways to support strategic networks when teaching mathematics.

– **Provide varied ways to interact and respond**

Students can demonstrate what they have learned in many ways other than traditional pencil-and-paper testing. Open-ended assignments, such as creating a game and determining its rules, allow students to apply knowledge in a new way. Students may participate in projects or create simulations, with each student taking responsibility for a part.

It is important that the method chosen for demonstrating knowledge does not pose its own problems. For example, a student who has trouble with handwriting might be more successful explaining answers orally, rather than in writing. Similarly, a student who becomes anxious when talking in class could write his explanations. Special consideration must be taken for a student with a physical disability that interferes with writing or speaking.

– **Provide scaffolds for practice and performance**

Many processes that are a part of mathematical thinking can be modeled for students. For example, a teacher may use "think aloud" to describe the reasoning process used in deciding what method of estimation would be best for a specific problem. Alternatively, this reasoning can be given to students in text and illustrations. For skills that should be routine or automatic, such as applying an algorithm, the steps can be posted in the room as a reference. Students also may be given worked-out examples of problems that they can use as a reference.

Many skills in mathematics, such as recalling basic facts, the steps in algorithms, rules for estimation, creating data displays, or using a calculator, need to be routine or automatic in order to be used effectively. If a student puts extra effort into any of these, then less effort is available for the higher-order thinking necessary for problem-solving. Practicing, however, does not mean merely rote drill. Supported practice helps students place the skill in a context so that they can know when and how to draw on this skill, both in the problems presented in mathematics classes and in everyday life.

– **Provide appropriate tools**

Another consideration is the supports a student may need when a skill is not automatic. For example, many students can apply their conceptual understanding of multiplication to solve relatively complex problems even though they may not remember all the steps of the multiplication algorithm. If they are required to do the calculations, most of their energy will be devoted to remembering these steps, rather than to analyzing the problem and applying their conceptual knowledge. Providing support for the calculations, such as using a calculator or working with a student who is facile with computation, will ensure that such students have ample opportunities to develop and refine their problem-solving skills.

– **Support planning and strategy development**

Present situations that require the student to select, apply, and adapt strategies to solve a novel problem, one in which the solution is not obvious or for which there can be more than one answer. These types of problems encourage students to reflect on the problem-solving process they are using and whether or not it is effective. Provide situations in which more than one approach will work and ones in which the student must reflect on progress and make adjustments. The goal of these opportunities is not necessarily to be correct but to learn to apply their skills effectively, to reflect on their process as they work, and to see alternative approaches that may also be appropriate.

– **Facilitate managing information and resources**

Often math problems are complex, requiring several steps and the need to draw on and keep track of information from several sources. Many times students do not have good strategies for keeping information in an organized manner so that they can use it efficiently. They can benefit from supports such as graphic organizers to serve this purpose. Tables and graphs can also be useful in organizing information; students need to learn the benefits of different formats of these tools.

– **Enhance capacity for monitoring progress**

Understanding number concepts and their applications is enhanced when students are helped to analyze errors and find ways to correct them. Students should be encouraged to question their own work and to find multiple solutions to problems. Teachers also need to be aware of situations in which students need extra support because of limitations in background knowledge or necessary procedures.

Designing Instruction to Support Affective Networks: Multiple Means of Engagement

The affective networks are essential structures in determining what is significant or important to an individual. These structures are critical in determining whether an object or situation is something to fear, something to crave, something to investigate, or something to ignore. The normal functioning of these structures thus results in the "why" of behavior, why we do what we do, or what we call motivation (Damasio, 1994; LeDoux, 1996).

What do these affective networks have to do with learning mathematics? They are not central either in recognizing the patterns of math or in performing its operations. They are central, however, in whether one engages in mathematics at all. At any given moment, there are many stimuli competing for attention and many possible actions to take. Whether a child will pay attention to the symbols of mathematics, or to the kinds of problem solving that mathematics entails, will depend in large part on the relative value (or importance) placed on those stimuli by the affective networks.

Mathematics presents a wide variety of situations that may create anxiety and avoidance in students (Gomez-Chacon, 2001). Remembering the steps of an algorithm may prove stressful for a student with memory problems (Bryant, Kim, Hartman, & Bryant, 2006). The emphasis on communication of mathematical ideas creates a difficult situation for students with language disabilities and for those who are anxious about speaking in front of their peers (Baxter, Woodward, & Olson, 2001). Good problem-solvers need to feel comfortable trying out different approaches to a problem, knowing there may not be only "one right way." Many students develop the idea that they are "just not good at math." Good teaching is largely the art of engagement, of finding what will motivate a child to learn mathematics and to feel confident in his or her ability in math.

Several of the UDL guidelines for affective networks relate to mathematics.

– **Offer choices**

Providing students with choices of content and tools can increase their interest in and enthusiasm for learning particular concepts and skills. The opportunity to link current learning to areas of particular interest can make learning easier. Students also are more likely to practice skills when they are embedded in activities that they enjoy.

– **Enhance relevance, value, and authenticity; reduce threats, distractions**

Many students enjoy the types of multimedia presentations offered by many software programs, but there are some who find this context overwhelming. Students need to have some control over the sights and sounds of their learning environment whenever possible.

Many students feel they cannot succeed at mathematics because they have trouble remembering the number facts and algorithms that are needed to find these types of answers. Other students enjoy the preciseness of facts and algorithms. They are uncomfortable in the more open-ended, problem-solving aspect of mathematics, where there may be more than one correct answer or several ways to find the answer. Both types of students need to be supported in the classroom.

Ideally, the rewards of learning mathematics should be intrinsic to the subject. However, in many situations, we need to provide external rewards. Embedding the learning in a game or puzzle format is one way to do this. You also can offer prizes as a reward for student work. Keep in mind that the ultimate goal should be to move toward intrinsic rewards whenever possible.

– **Vary levels of challenge and support**

.Students need tasks that are challenging; not so easy that they become boring nor so difficult that they are viewed as requiring too much effort. This optimal level of challenge varies from student to student; and it varies for individual students, depending on the task, the context, and other factors not directly related to learning (such as concerns at home). Some students may feel comfortable moving forward in small steps, with frequent opportunities to practice what they are learning. Others enjoy the challenge of a larger, open-ended learning situation. Adjustable levels of challenge will allow both of these groups of students to work at their optimal level of challenge.

– **Increase mastery-oriented feedback**

Feedback is essential to the development of problem-solving skills. Use the language of mathematics when providing feedback so that students learn to use communication as a strategy for reflection on their work. Help students pinpoint problems in their thinking process and alternative approaches to try. Help them make judgments about the reasonableness of their solutions. Encourage students to evaluate the mathematical thinking and strategies of others. Feedback is also necessary when students are learning the skills that need to become routine or automatic. For the feedback to be relevant, it should not merely indicate whether or not the student has performed correctly but should help the student see what to change in order to be correct. For example, many students make errors in subtraction because of problems in regrouping. Feedback for these students might include how regrouping is related to place value.

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